Assembly of Element Matrices

$$[k]^e = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global	[1]	[2]		
Local	[1]	[2]		
1		−1	[1]	[1]
−1		1	[2]	[2]

[1]	[7]		
[1]	[2]		
1	−1	[1]	[1]
−1	1	[2]	[7]

Global	[2]	[3]		
Local	[1]	[2]		
1	−1		[1]	[2]
−1	1		[2]	[3]

[2]	[6]		
[1]	[2]		
1	−1	[1]	[2]
−1	1	[2]	[6]

Effect of Node Numbering on System Matrices

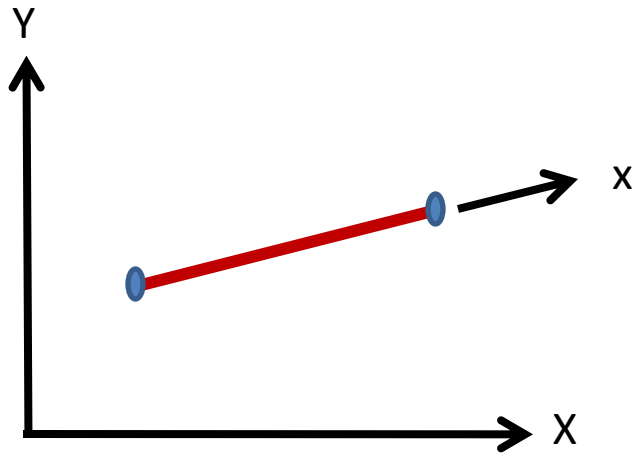
$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 2 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 2 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \\ 2 & -1 \\ 2 & -1 \\ 2 & -1 \\ 2 & -1 \\ 1 & 0 \end{bmatrix}$$

Assembly – nodal dof are vectors!

- Temperatures are scalar quantities
- So direction independent
- Element matrices or equations wont change if coordinate frame is changed
- Such scalar problems – element matrices can be directly assembled
- In some problems eg. Structural displacements are vectors and hence direction dependent
- Need to transform all of the dof into one common global reference coordinate frame



$$\frac{AE}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix}$$

$$u_i = u_{iX}\cos\theta + u_{iY}\sin\theta$$

$$u_j = u_{jX}\cos\theta + u_{jY}\sin\theta$$

$$\begin{Bmatrix} u_i \\ u_j \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} u_{iX} \\ u_{iY} \\ u_{jX} \\ u_{jY} \end{Bmatrix}$$

$$\{\delta\}_\ell^e = [T]\{\delta\}_g^e$$

$$[k]_\ell^e \{\delta\}_\ell^e = \{F\}_\ell^e \qquad [k]_\ell^e [T]\{\delta\}_g^e = \{F\}_\ell^e$$

$$\left([T]^T [k]_\ell^e [T] \right) \{\delta\}_g^e = [T]^T \{F\}_\ell^e$$

$$[T]^T \{F\}_\ell^e = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & \cos\theta \\ 0 & \sin\theta \end{bmatrix} \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} = \begin{Bmatrix} F_i \cos\theta \\ F_i \sin\theta \\ F_j \cos\theta \\ F_j \sin\theta \end{Bmatrix}$$

$$[k]_g^e \{\delta\}_g^e = \{F\}_g^e$$

$$[k]_g^e = [T]^T [k]_\ell^e [T] \qquad [T]^T \{F\}_\ell^e = \{F\}_g^e$$

$$[k]_\ell^e = \frac{AE}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global	$(1X)$	$(1Y)$	$(3X)$	$(3Y)$		
Local	(iX)	(iY)	(jX)	(jY)	*Local*	*Global*

$$[k]_g^{(1)} = \frac{AE\cos\alpha}{L}\begin{bmatrix} \cos^2\alpha & \sin\alpha\cos\alpha & -\cos^2\alpha & -\sin\alpha\cos\alpha \\ & \sin^2\alpha & -\sin\alpha\cos\alpha & -\sin^2\alpha \\ & & \cos^2\alpha & \sin\alpha\cos\alpha \\ \textit{Sym.} & & & \sin^2\alpha \end{bmatrix} \begin{matrix} (iX) & (1X) \\ (iY) & (1Y) \\ (jX) & (3X) \\ (jY) & (3Y) \end{matrix}$$

Summary : General Method of Assembly

Step 1: Identify the local (i.e. element level) and global (i.e. structure level) coordinate frames of reference.

Step 2: Obtain the element matrices in the local reference frame.

Step 3: Obtain the coordinate transformation matrix [*T*] between the local and global frames of reference.

Step 4: Transform the element matrices into the common global reference frame.

Step 5: Identify the global locations of the individual coefficients of the element matrices based on local and global node numbers.

Step 6: Assemble the element matrices by placing the coefficients of the element matrices in their appropriate places as identified in Step 5.

BEAM ELEMENT – Fourth Order D.E

$$EI \frac{d^4 v}{dx^4} - q_0 = 0$$

$v(0) = 0,$ $\quad v(L) = 0,$

$$\frac{d^2 v}{dx^2}(0) = 0 \qquad \frac{d^2 v}{dx^2}(L) = 0$$