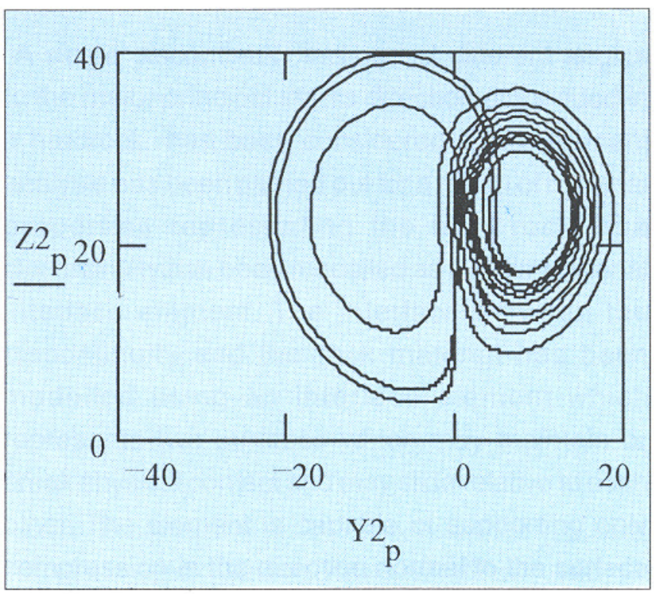
4. NONLINEAR DYNAMICAL SYSTEMS

4.1 A New Method to Embed Time Series Data

We have demonstrated how to calculate the derivatives of a discretely sampled data, far more accurately than the usual central difference formulae do. This was accomplished by setting the Taylor's series formulation in a matrix form and then using singular value decomposition. These derivatives are used to generate a set of vectors from scalar data. This method has certain advantages over the conventional method of embedding data. Using the example of a numerically generated data, it has been shown how to obtain state space portraits from sampled data (for example, see *Fig. 4.1.1* which shows the results for the Lorenz data). It is also shown that this method is robust in the sense that it can tolerate and in fact, suppress a small amount of random noise (as shown in *Fig. 4.1.2*).

(P.G. Vaidya)

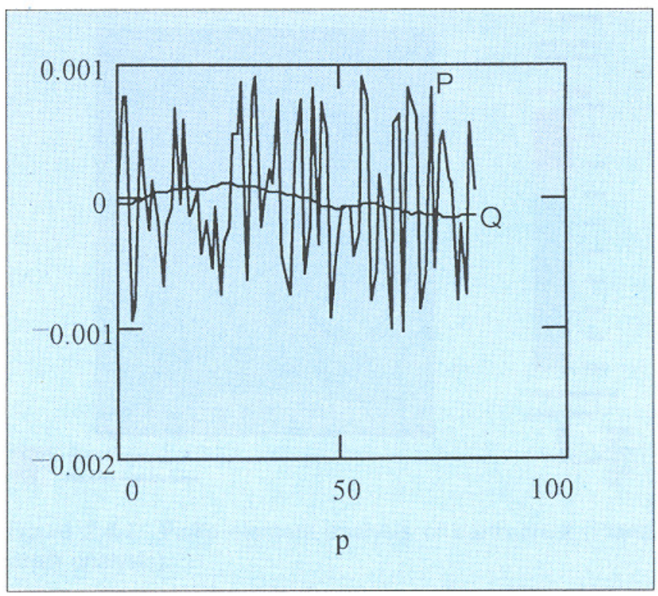
Fig. 4.1.1 Lorenz's equaton is usually stated in terms of the variables X, Y and Z. This figure shows a significantly improved technique of finding alternative state space variable Y2 and Z2 from X alone, (without any additional knowledge about the equation or its parameters). These variable are homeomorphs of the usual Y and Z and are an example of a multitude of alternative phase space representations (p-time step).



4.2 Cryptology Based on Chaotic Synchronization

There are two properties of chaos, which make it an attractive medium for cryptology. The first of these is well known: a chaotic signal shares many properties with random signals. This makes it possible to use a chaotic signature to generate uncorrelated keys for encryption. The second property (discovered only in this decade) is that some chaotic equations can be synchronized with others, by passing only partial information from one to another. We have shown that the synchronization does not work if the parameters in the equation are wrongly guessed and that these properties can be

Fig. 4.1.2. The P curve represents the noise added to the signal and the Q curve shows the remaining noise after the advanced embedding procedure has been carried out. The significant reduction in noise is a side benefit of the procedure (p-time step).



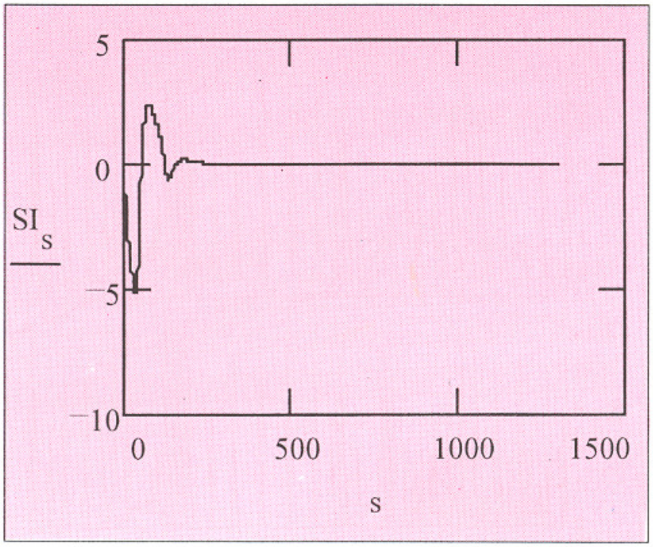


Fig. 4.2.1. The synchronization index (SI_p) at each step (p). When this index is acceptably close to zero, we know that the synchronization is taking place. This has been used to speed up the "hook up" of the two systems.

used to encrypt and decode messages. Recently, using the ideas of the earlier section (4.1), we have developed a technique to calculate the synchronization index (SI), which tells the receiver if synchronization has indeed taken place, and to speed up the synchronization so that the messages can be speeded up. *Fig. 4.2.1* above shows the convergence of the synchronization index. Once it gets below a certain threshold value, error free communication becomes possible.

(P.G. Vaidya)

4.3 Identifying Unknown Parameters in a Dynamical System

Another recent development deals with the possibility of rapidly identifying parameters of, for example, the Lorenz equation (β,ρ). It has been shown, that the various state space derivatives (P, Q, R, S) of the data bear the following relationship:

$$\left[\beta\cdot\frac{(-R-Q)}{P}-P\cdot Q+\frac{(-P\cdot(S+R)+Q\cdot(R+Q))}{P^2}\right]S$$
$$+\beta\left(\rho-1-\frac{Q}{P}\right)+\frac{(-P\cdot R+Q^2)}{P^2}-P^2=0$$

Using the method referred to in the equation alongside, we can accurately find P, Q, R and S and then using a simple nonlinear solver find the parameters. This is quite useful in many dynamical systems applications, including cryptanalysis.

(P.G. Vaidya)

4.4 Synchronisation of Chaotic Meta-populations in a Cascade of Coupled Patches

The aspects of nonlinear dynamical systems and chaotic dynamics have a lot of potential applications not only in physics but also in various other branches of science such as chemistry, biology, ecology, etc. Here we apply one such aspect of synchronisation of chaos to ecological modelling especially in studying the population dynamics of a cascade of coupled patches.

Most populations in nature are made up of a number of isolated patches of subpopulations which are subjected to migration. Migration is a common ecological process by which the size of the meta-population in different patches may change. Much attention has recently been focused on studying such spatial effects, like migration, on population dynamics. The dynamics of metapopulations in two patches undergoing migration among each other has commonly been analysed by using two coupled logistic map models (*Fig.4.4.1a*). Considering two sets of such coupled patches which are all exhibiting chaotic oscillations, we have shown that the chaotic metapopulations of the second set of patches X_2, Y_2 (response system) can be synchronised with that of the first set of patches X_1, Y_1 (drive system) by using a simple synchronisation technique which requires a selective migration only from the drive system patches (*Fig. 4.4.1b*).

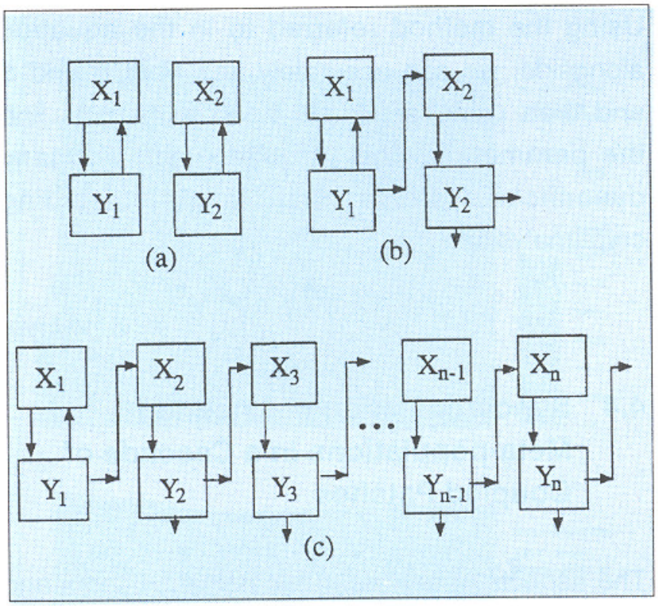


Fig. 4.4.1. Coupled logistic map models.

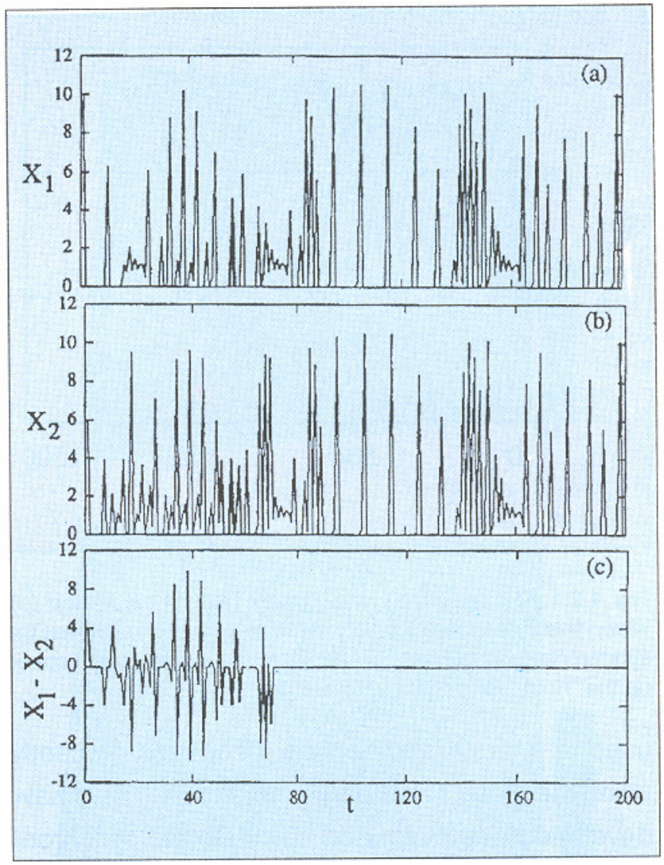


Fig. 4.4.2. Evolution of metapopulation.

We have used a simple variant of the Pecora and Carroll synchronisation technique for this purpose. After synchronisation, the populations of the response system patches exhibit chaotic oscillations which are exactly similar to that of the drive system patches, irrespective of their initial population sizes. *Fig.4.4.2* shows that the chaotic metapopulations of the response system patch X_2 (see *Fig. 4.4.2b*) is completely synchronised with that of the drive system patch X_1 (see *Fig. 4.4.2a*) after a short initial transient. The difference ($X_1 - X_2$) in the metapopulations of the two patches at every generation becomes zero after a few generations, as shown in *Fig. 4.4.2c*, which means that the chaotic metapopulations attain synchronisation resulting in identical chaotic oscillations in both the patches X_1 and X_2. It is interesting to note that the metapopulations in the other patches Y_1 and Y_2 are also synchronised simultaneously and exhibit similar chaotic oscillations.

Further, it has been shown that not only can two such sets of coupled patches be synchronized but also cascading of many such sets of coupled patches, in a similar way (*Fig. 4.4.1c*), results in synchronisation of their common signals. The synchronisation of chaotic metapopulations has an important implication in that it allows one to study the dynamics of chaotic metapopulations of a collection of many sets of coupled patches by simply analysing the behaviour of one set of patches alone. Finally, it is also observed that addition of small external noise can be effective in synchronisation when slight inhomogeneities in the environment affect the synchronisation procedure, and is therefore more biologically realistic.

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4.5 Experimental Forecasts of All-India Monthly and Summer Monsoon Rainfall Using Neural Network

Continuing our efforts at long-range forecast of rainfall patterns, the neural network forecast of all-India summer monsoon rainfall (ISMR) is being extended to forecast an all-India monthly rainfall (AIMR). As in the case of ISMR, the basic tool used is the cognitive network (CN) developed at C-MMACS. Since the development of the cognitive network, we have followed a use-and-probe strategy with the neural network (NN) forecast toll. In particular, we believe that while many of the shortcomings of NN in general and cognitive network in particular, need to be addressed at conceptual and theoretical levels, the capabilities and the scope of applicability of CN as a forecast tool also need to be explored. These two efforts can, and should, feedback upon each other, leading to better understanding and further development.

With this philosophy in the background, we have been generating experimental forecasts of ISMR for the last four years. It is noteworthy that all the three forecasts for 1995, 1996 and 1997 were generated well ahead of the monsoon season, and were found to be of good quality. The CN forecast for 1997 ISMR was, for example, 945mm (or 101% of long term mean) while the observed value was about 102%. The experimental forecast for 1998 is 945mm (i.e. 107%). The forecast for 1998 assumes further significance since it is being made in 1996, two years in advance. These successes along with the hindcast skill achieved during our investigation suggest it to be worthwhile to pursue this approach. The work is now in progress to extend the methodology to higher temporal and spatial resolutions.

(P. Goswami)